

Warm-Up

1. If \$5000 is invested into an account with an APR of 3.5%, how much will the account be worth in 7 years if it is compounded....

- a) Annually $n=1$
b) Weekly $n=52$
c) Continuously

$$A = P(1 + \frac{r}{n})^{nt}$$

$$\$6,361.40$$

$$\$6,387.58$$

$$\$6,388.11$$

$$A = P(1+r)^t \quad A = P(1-r)^t \quad A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

Logarithmic Functions

Review Inverse Functions

$$f(x) = x^3 + 4$$

$$y = x^3 + 4$$

$$x = y^3 + 4$$

$$f^{-1}(x) = \sqrt[3]{x-4}$$

$$\sqrt[3]{x-4} = y^3$$

$$\sqrt[3]{x-4} = y$$

$$f(x) = 2^x$$

$$y = 2^x$$

$$x = \log_2 y$$

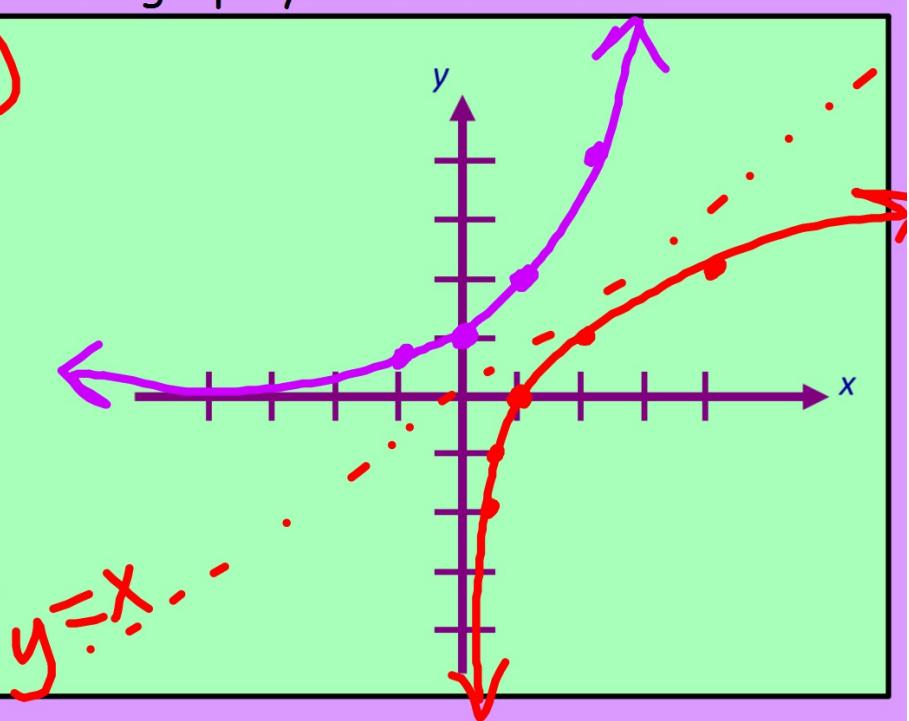
$$f^{-1}(x) =$$

Recall: To find an inverse of a function, you switch the x and y values. Create a table of values for $y = 2^x$. Use the table of values to graph y and its inverse.

$$f(x) = 2^x \quad f^{-1}(x)$$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



Definition of Logarithmic Function with Base a

For $x > 0$, $a > 0$, and $a \neq 1$.

$$y = \log_a x \text{ if and only if } x = a^y$$

The function given by

$$f(x) = \log_a x \text{ (read this as "log base a of x")}$$

is called the Logarithmic Function with Base a

What you need to understand about Logarithmic Functions

- Inverse of Exponential Function

:

- For Logs, $x > 0$ $B > 0$ and $B \neq 1$

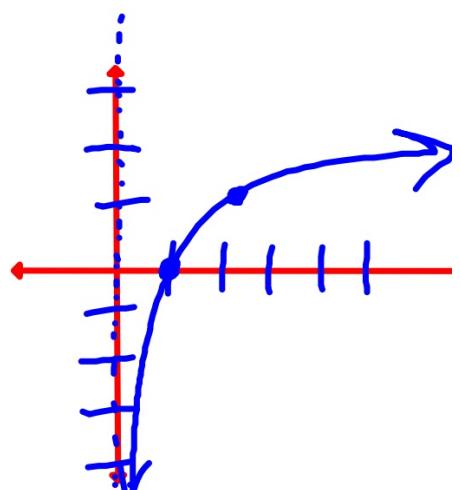
$$y = \log_B x$$

Graph of Parent Logarithmic Functions

$$f(x) = \log_B x$$

$$\begin{aligned} &\log_2 x \\ &\log_3 x \end{aligned}$$

- Vertical Asymptote at $x=0$
- X-Intercept: $(1, 0)$
- Y-Intercept: None
- Point at $(B, 1)$
- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$



Transformations of Parent Graphs

$$f(x) = -4 \log_B(-x + 4) - 7$$

Handwritten annotations explaining transformations:

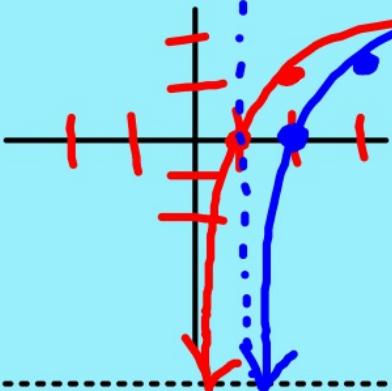
- Reflection about x-Axis (yellow arrow)
- Stretch/Shrink (green arrow)
- Horizontal Shift left 4 (red arrow)
- Vertical shift down 7 (blue arrow)

Shifting Graphs of Logarithmic Functions

Parent Graph:
 $f(x) = \log_2 x$

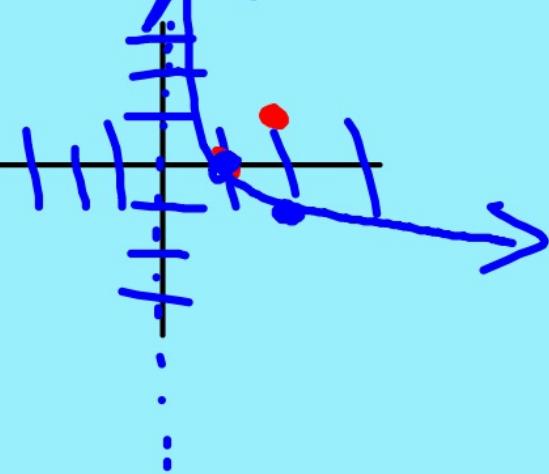
$$g(x) = \log_2(x - 1)$$

Transformation: H.S. Right



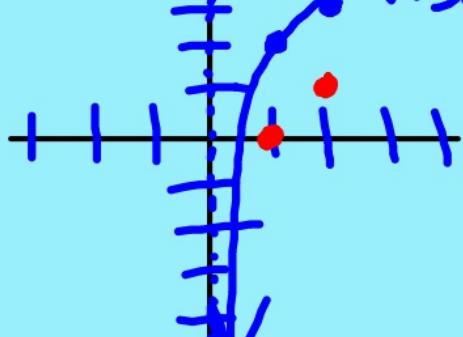
$$j(x) = -\log_2(x)$$

Transformation:



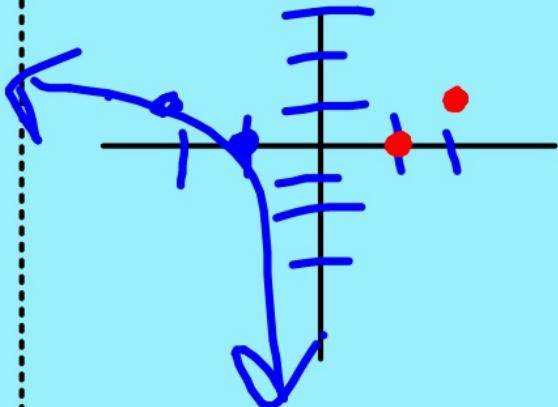
$$h(x) = \log_2(x) + 2$$

Transformation: V.S. up 2



$$k(x) = \log_2(-x)$$

Transformation:



Converting between Logs and Exponentials

$$\log_B x = y \quad \longleftrightarrow \quad B^y = x$$

$$\log_B x = y \iff B^y = x$$

Examples: Converting Logarithms to Exponentials

$$1) \log_2 16 = 4$$
$$2^4 = 16$$

$$2) \log_3 81 = 4$$
$$3^4 = 81$$

$$3) \log_7 343 = 3$$
$$7^3 = 343$$

$$4) \log_4(1/16) = -2$$
$$4^{-2} = \frac{1}{16}$$

$$\text{Log}_B X = Y \Leftrightarrow B^Y = X$$

Examples: Converting Exponentials to Logarithms

$$1) \quad 5^3 = 125$$

$$\log_5 125 = 3$$

$$2) \quad 2^5 = 32$$

$$\log_2 32 = 5$$

$$3) \quad 4^3 = 64$$

$$\log_4 64 = 3$$

$$4) \quad 3^{-2} = 1/9$$

$$\log_3 (1/9) = -2$$

$$\log_B X = y \Leftrightarrow B^y = X$$

Tip: Notice the "base" of the exponent.

Logarithms that appear to have no base

- $\log 3$ is really $\log_{10} 3$
- Logarithms with base 10 are called the Common logs
- Calculators are designed to evaluate these specifically

Evaluating Logarithms

$$f(x) = \log x$$

$$1) f(10) = \log_{10}(10) = 1$$

$$2) f(2.5) = \log_{10}(2.5) = 0.40$$

$$3) f(-2) = \log_{10}(-2) = \text{Nonreal Answer}$$

Evaluating Logarithms

1) $f(x) = \log_2 x$ find $f(64)$

$$\log_2 64 = z$$

$$2^z = 64$$

$$f(64) = 6$$

2) $f(x) = \log_7 x$ find $f(1)$

$$\log_7 1 = R$$

$$7^R = 1$$

$$f(1) = 0$$

3) $f(x) = \log_4 x$ find $f(2)$

$$\log_4(2) = R$$

$$4^R = 2$$

$$f(2) = \frac{1}{2}$$

Properties of Logs

$$1) \log_4 1 = \underline{\quad}$$

$$\log_4 1 = R$$

$$4^R = 1$$

General Rule

$$\log_B 1 = \underline{\quad}$$

$$2) \log_3 3^1 = \underline{\quad}$$

$$\log_3 3^R = R$$

$$3^R = 3$$

$$3) \log_8 8^4 = \underline{\quad}$$

$$\log_8 (8^4) = R$$

$$8^R = 8^4$$

General Rules

$$\log_B B^R = R$$

$$4) 6^{\log_6 20} = \underline{20}$$

$$6^{\log_6 R} = R$$

$$\log_6 R = \log_6 20$$

$$5) 10^{\log_{10} 5} = \underline{5}$$

$$10^{\log_{10} R} = R$$

$$\log_{10} R = \log_{10} 5$$

General Rule

$$B^{\log_B R} = R$$

$$5) \log_5 125^x = \underline{3x}$$

$$\log_5 125^x = R$$

$$5^R = 125^x$$

$$5^R = (5^3)^x$$

$$5^R = 5^{3x}$$

$$6) \log_3 9^x = \underline{2x} .$$

$$3^R = 9^x$$

$$3^R = (3^2)^x$$

$$3^R = 3^{2x}$$

